Go Directly to Jail

Monopoly, but fun.

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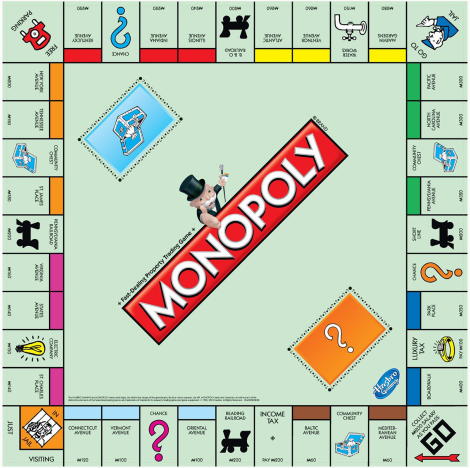
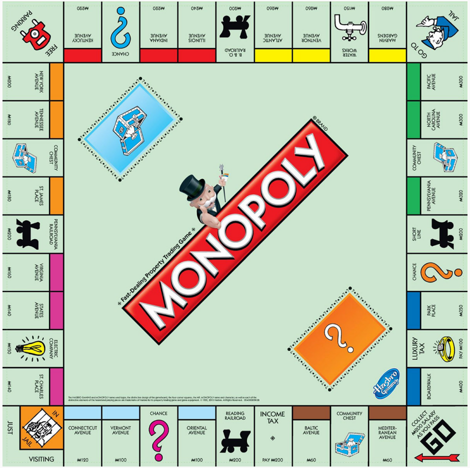
ECON–101



Monopoly is a game of acquisition.

Players need property, but have limited funds.

Which properties should players invest in?

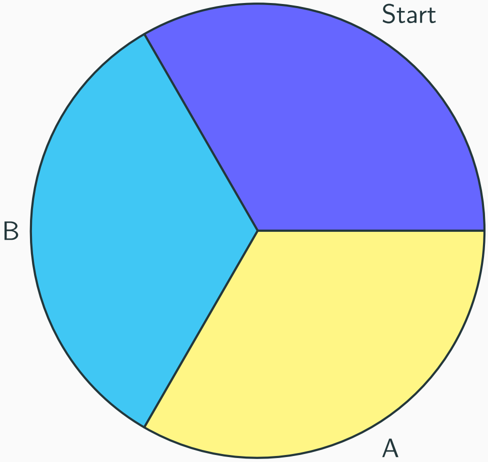


Value depends on more than price.

You have to consider *location*.

Does every space occur equally often?

A simpler game



Consider the following game:

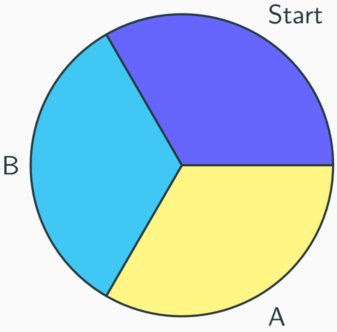
- Flip a coin.
- If it's heads, advance once.
- If it's tails, advance twice.

Definition

Steady states

If, as a game goes on forever, the probability that we are in any particular space approaches a constant, then the list of all such probabilities is the *steady state* of the game.

Steady state of the previous game:



State	Probability
Start	$1/3$
A	$1/3$
B	$1/3$

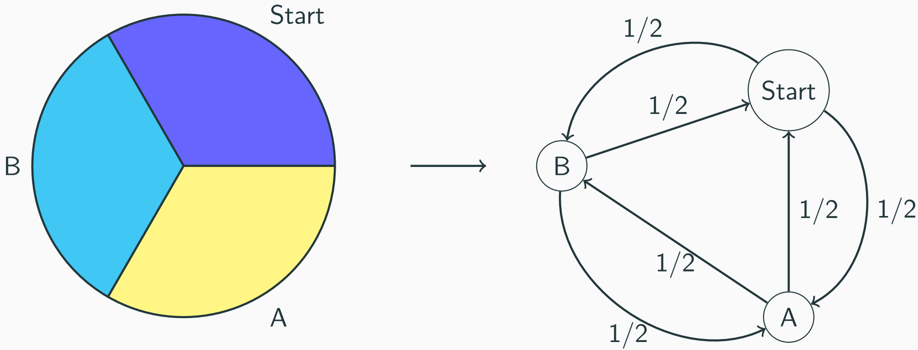
Markov chains

We compute steady states using a tool from probability theory, *Markov chains*:

- Invented in the early twentieth century by Andrey Markov, a Russian mathematician and early pioneer in probability theory.
- Models systems of states that are memoryless. (The next state depends only on the current state.)
- Can model gambling, the movement of populations, the occurrence of words in corpora of literature, and so on.

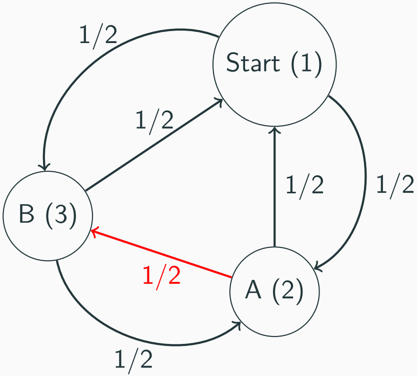
How do they work? (I)

A system is translated into a set of states, which are assigned *transition probabilities*.



How do they work? (II)

These transition probabilities are placed into the *transition matrix* $P = (p_{ij})$, where p_{ij} is the probability of transitioning from state j to state i. (Each state gets its own *column*.)



$$\longrightarrow \qquad P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & \color{red}{1/2} & 0 \end{bmatrix}.$$

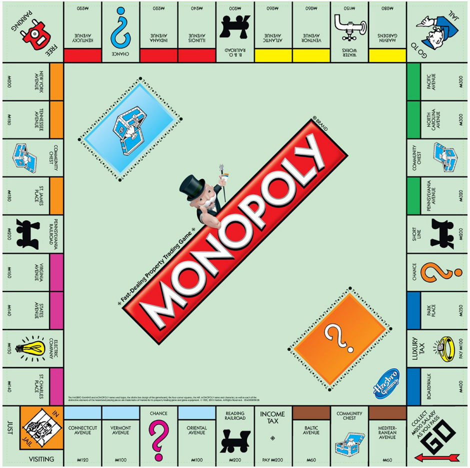
How do they work? (III)

Computing steady states amounts to computing *eigenvectors* of the transition matrix.

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \qquad \longrightarrow \qquad \mathbf{v} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

(This is a standard linear algebra computation.)

Back to Monopoly



The same process for *Monopoly* will determine the most popular properties.

The procedure is identical, but bigger.

Monopoly matrix

$$
P = \left[\begin{array}{cccccccccccccccccccccccccccccccccccccccccc}
\frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{67}{2304} & \frac{329}{5760} & \frac{323}{2880} & \frac{1609}{11520} & \frac{107}{640} & \frac{1603}{11520} & \frac{1283}{11520} & \frac{161}{1920} & \frac{649}{11520} & \frac{83}{2880} & \frac{1}{768} & 0 & 0 & 0 \\
\frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{169}{5760} & \frac{27}{320} & \frac{1289}{11520} & \frac{803}{5760} & \frac{641}{3840} & \frac{1603}{11520} & \frac{643}{5760} & \frac{323}{3840} & \frac{163}{2880} & \frac{67}{2304} & 0 & 0 & 0 \\
\frac{169}{5760} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{163}{2880} & \frac{323}{3840} & \frac{643}{5760} & \frac{1603}{11520} & \frac{641}{3840} & \frac{803}{5760} & \frac{1289}{11520} & \frac{27}{320} & \frac{131}{2304} & 0 & 0 & 0 \\
\frac{329}{5760} & \frac{67}{2304} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{83}{2880} & \frac{649}{11520} & \frac{161}{1920} & \frac{1283}{11520} & \frac{1603}{11520} & \frac{107}{640} & \frac{1609}{11520} & \frac{323}{2880} & \frac{65}{768} & 0 & 0 & 0 \\
\frac{163}{1920} & \frac{131}{2304} & \frac{83}{2880} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{329}{11520} & \frac{323}{5760} & \frac{107}{1280} & \frac{1283}{11520} & \frac{803}{5760} & \frac{643}{3840} & \frac{403}{2880} & \frac{259}{2304} & 0 & 0 & 0 \\
\frac{649}{5760} & \frac{65}{768} & \frac{163}{2880} & \frac{329}{11520} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{163}{5760} & \frac{643}{11520} & \frac{107}{1280} & \frac{643}{5760} & \frac{1609}{11520} & \frac{161}{960} & \frac{323}{2304} & 0 & 0 & 0 \\
\frac{809}{5760} & \frac{259}{2304} & \frac{27}{320} & \frac{649}{11520} & \frac{163}{5760} & \frac{1}{3840} & 0 & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{323}{11520} & \frac{643}{11520} & \frac{161}{1920} & \frac{1289}{11520} & \frac{403}{2880} & \frac{43}{256} & 0 & 0 & 0 \\
\frac{203}{1920} & \frac{203}{2304} & \frac{203}{2880} & \frac{203}{3840} & \frac{203}{5760} & \frac{203}{11520} & 0 & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{203}{11520} & \frac{203}{5760} & \frac{203}{3840} & \frac{203}{2880} & \frac{203}{2304} & 0 & 0 & 0 \\
\frac{809}{5760} & \frac{43}{256} & \frac{403}{2880} & \frac{1289}{11520} & \frac{161}{1920} & \frac{643}{11520} & \frac{1}{36} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{163}{5760} & \frac{649}{11520} & \frac{27}{320} & \frac{259}{2304} & 0 & 0 & 0 \\
\frac{649}{5760} & \frac{323}{2304} & \frac{161}{960} & \frac{1609}{11520} & \frac{643}{5760} & \frac{107}{1280} & \frac{1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{329}{11520} & \frac{163}{2880} & \frac{65}{768} & 0 & 0 & 0 \\
\frac{163}{1920} & \frac{259}{2304} & \frac{403}{2880} & \frac{643}{3840} & \frac{803}{5760} & \frac{1283}{11520} & \frac{1}{12} & \frac{1}{18} & \frac{1}{36} & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{83}{2880} & \frac{131}{2304} & \frac{1}{6} & \frac{1}{6} & 0 \\
\frac{329}{5760} & \frac{65}{768} & \frac{323}{2880} & \frac{1609}{11520} & \frac{107}{640} & \frac{1603}{11520} & \frac{1}{9} & \frac{1}{12} & \frac{1}{18} & \frac{1}{36} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{67}{2304} & 0 & 0 & 0 \\
\frac{169}{5760} & \frac{131}{2304} & \frac{27}{320} & \frac{1289}{11520} & \frac{803}{5760} & \frac{641}{3840} & \frac{5}{36} & \frac{1}{9} & \frac{1}{12} & \frac{1}{18} & \frac{323}{11520} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & 0 & 0 & \frac{1}{36} \\
\frac{1}{640} & \frac{67}{2304} & \frac{163}{2880} & \frac{323}{3840} & \frac{643}{5760} & \frac{1603}{11520} & \frac{1}{6} & \frac{5}{36} & \frac{1}{9} & \frac{1}{12} & \frac{643}{11520} & \frac{163}{5760} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & 0 & 0 & \frac{1}{18} \\
\frac{1}{640} & \frac{1}{768} & \frac{83}{2880} & \frac{649}{11520} & \frac{161}{1920} & \frac{1283}{11520} & \frac{5}{36} & \frac{1}{6} & \frac{5}{36} & \frac{1}{9} & \frac{107}{1280} & \frac{323}{5760} & \frac{329}{11520} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & 0 & 0 & \frac{1}{12} \\
\frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{329}{11520} & \frac{323}{5760} & \frac{107}{1280} & \frac{1}{9} & \frac{5}{36} & \frac{1}{6} & \frac{5}{36} & \frac{1283}{11520} & \frac{161}{1920} & \frac{649}{11520} & \frac{83}{2880} & \frac{1}{768} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & 0 & 0 & \frac{1}{9} \\
\frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{163}{5760} & \frac{643}{11520} & \frac{1}{12} & \frac{1}{9} & \frac{5}{36} & \frac{1}{6} & \frac{1603}{11520} & \frac{643}{5760} & \frac{323}{3840} & \frac{163}{2880} & \frac{67}{2304} & \frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & 0 & 0 & \frac{5}{36} \\
\frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{323}{11520} & \frac{1}{18} & \frac{1}{12} & \frac{1}{9} & \frac{5}{36} & \frac{641}{3840} & \frac{803}{5760} & \frac{1289}{11520} & \frac{27}{320} & \frac{131}{2304} & \frac{169}{5760} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & 0 & 0 & \frac{1}{6} \\
\frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{36} & \frac{1}{18} & \frac{1}{12} & \frac{1}{9} & \frac{1603}{11520} & \frac{107}{640} & \frac{1609}{11520} & \frac{323}{2880} & \frac{65}{768} & \frac{329}{5760} & \frac{67}{2304} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & 0 & 0 & \frac{5}{36} \\
\frac{1}{640} & \frac{1}{768} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & \frac{1}{36} & \frac{1}{18} & \frac{1}{12} & \frac{1283}{11520} & \frac{803}{5760} & \frac{643}{3840} & \frac{403}{2880} & \frac{259}{2304} & \frac{163}{1920} & \frac{131}{2304} & \frac{83}{2880} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & 0 & 0 & 0 & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & \frac{1}{640} & \frac{1}{960} & \frac{1}{1280} & \frac{1}{1920} & \frac{1}{3840} & \frac{1}{3840} & \frac{1}{1920} & \frac{1}{1280} & \frac{1}{960} & \frac{1}{768} & 0 & 0 & \frac{1}{9} \\
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\end{array}\right]
$$

Much bigger.

Most Likely

Table 1: Ten most likely spaces in *Monopoly*.

Space	Limiting Probability
Jail (10)	9.598
Community Chest (17)	2.726
Tennessee Avenue (18)	2.700
New York Avenue (19)	2.687
Free Parking (20)	2.683
Kentucky Avenue (21)	2.664
B. & O. Railroad (25)	2.654
Illinois Avenue (24)	2.650
Atlantic Avenue (26)	2.646
Indiana Avenue (23)	2.631

Least Likely

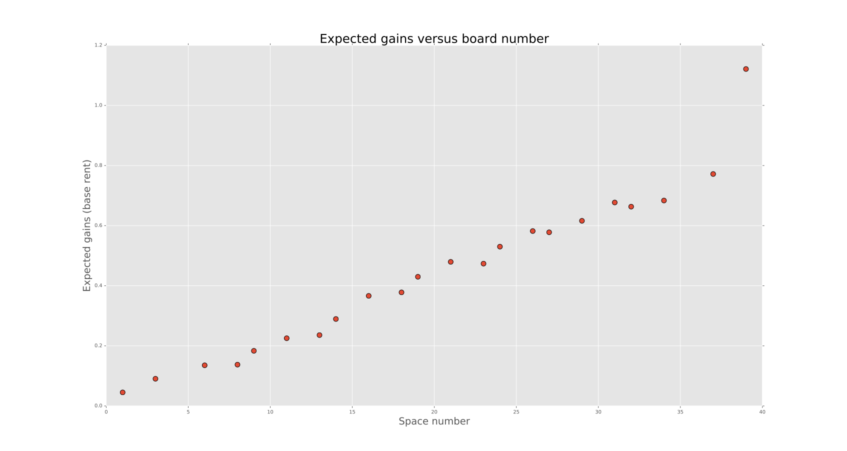
Table 2: Ten least likely spaces in *Monopoly*.

Space	Limiting Probability
Community Chest 1 (2)	2.257
Oriental Avenue (6)	2.255
Mediterranean Avenue (1)	2.254
St. Charles Place (11)	2.254
GO (0)	2.250
Income Tax (4)	2.248
Reading Railroad (5)	2.246
Boardwalk (39)	2.243
Luxury Tax (38)	2.229
Park Place (37)	2.205

Expected income

Not every space is equally likely, but it isn't clear that this is useful.

Expected income is hardly affected.



What does this mean?

- Do we have a new, winning *Monopoly* strategy?
  - Probably not: expected gain suggests that conventional wisdom is correct.
- Is *Monopoly* AI a lost cause?
  - No: it is susceptible to standard machine learning techniques, like reinforcement learning.
- Can we learn any larger lessons?
  - Yes: spaces are not uniformly-distributed. Conventional wisdom isn't rigorous. Markov chains are cool.

Questions?

(For more, see: Abbot and Richey. "Take a walk on the boardwalk," The College Mathematics Journal)

rwbogl/boardwalk