The Number Two Does Not Exist

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Misconceptions

These are the natural numbers:

$$\{1, 2, 3, 4, \ldots\}$$

Misconceptions

These are the natural numbers:

$$\{1, 2, 3, 4, \ldots\}$$

For the next fifteen minutes, the natural numbers look like this:

$$\{1, \quad 3, 4, \ldots\}.$$

Motivations

- Is this allowed? (Yes)
- Why are we doing this? (I'll tell you later)

Mo' definitions

Prime numbers

A natural number $n \neq 1$ is *prime* provided that its only factors are 1 and itself. A number that is not prime is called *composite*.

The numbers 3, 5, and 7 are prime.

The numbers $4 = 2 \cdot 2$, $6 = 2 \cdot 3$, and $14 = 2 \cdot 7$ are composite.

The primes are sometimes called the "building blocks" of the natural numbers.

The number 2 *was* prime. Did getting rid of it change anything?

(Think about Jenga.)

Primes without 2

What about 4?

$$4 = 2 \cdot 2$$

There isn't a 2 anymore...

Others

$$4 = 2 \cdot 2$$
$$6 = 2 \cdot 3$$
$$8 = 2 \cdot 2 \cdot 2$$
$$10 = 2 \cdot 5$$

n	old prime?	new prime?
1		
3	✓	✓
4		✓
5	✓	✓
6		✓
7	✓	✓
8		✓
9		
10		✓

The Sieve of Eratosthenes

Is there anyway that we could visualize the new primes?

If only someone had created an animation to do that...

Distribution of the Primes

The Prime Counting Function

The prime counting function, denoted $\pi(x)$, counts the number of primes less than or equal to x.

$$\pi(10) = 4 \quad (2, 3, 5, 7)$$

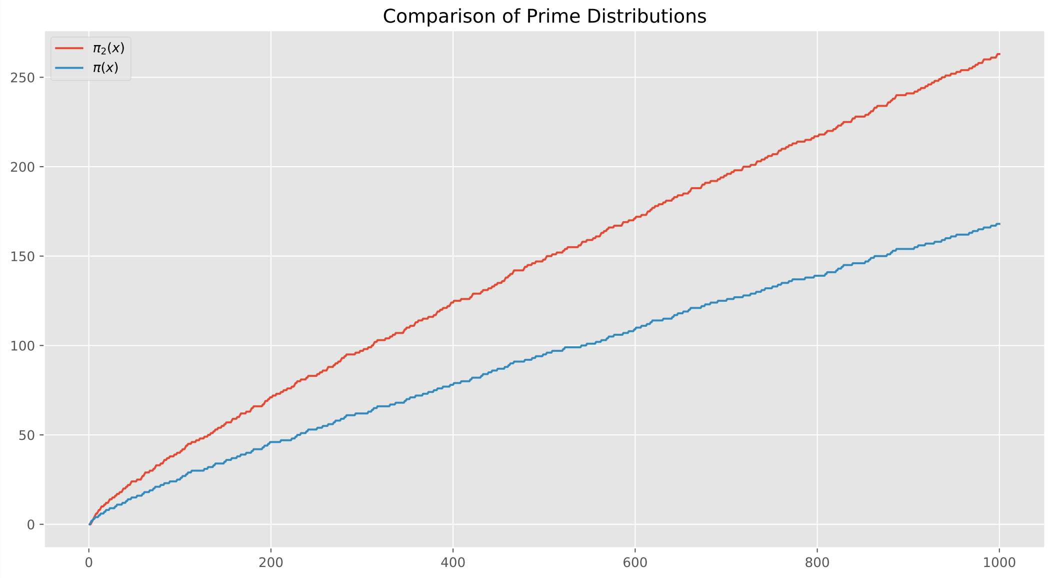
The New Prime Counting Function

The new prime counting function, denoted $\pi_2(x)$, counts the number of primes less than or equal to x, *after we remove the number* 2.

$$\pi_2(10) = 7 \quad (3, 4, 5, 6, 7, 8, 10)$$

(This is *not* the number $\pi = 3.14159\ldots$, it just uses the same Greek letter.)

Prime Distributions



The tower wobbles...

Unique Prime Factorization

Every natural number can be factored into a *unique* product of primes.

$$
\begin{aligned}
4 &= 2 \cdot 2 \\
6 &= 2 \cdot 3 \\
8 &= 2 \cdot 2 \cdot 2 \\
12 &= 2 \cdot 2 \cdot 3
\end{aligned}
$$

Does this still work?

... and comes crashing down

Think about 64:

$$64 = 8 \cdot 8$$

$$64 = 4 \cdot 4 \cdot 4$$

Prime factorizations are no longer unique.

Justification

Again: Why do this?

Boring answer: Just a puzzle.

Better answer:

- Everyone uses primes (mathematics and internet security)
- If we rely on them, we should be able to answer "silly questions."
- If we can't, we don't know enough to use them.

Questions?