Contaminants in a Mother–Fetus System

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The model used by Verner et al. [2013] does not formally appear in their paper. However, we can essentially reconstruct it by analyzing their published `acslX` code. What follows is a discussion of a slightly simplified version of this model.

Important variables:

- $M(t)$: amount of contaminants in mother
- $C(t)$: amount of contaminants in child
- $L(t)$: rate at which contaminants exit through milk lipids in breast milk (determined by interpolation of a table and data determining proportion of food attributable to breastmilk at time t)
- $B(t)$: bodyweight of mother (determined by linear interpolation of a table)
- $V_M(t)$: mother body fat volume (determined by linear interpolation of a table)
- $V_C(t)$: child body fat volume (determined by linear interpolation of a table)
- d: dosing of contaminants the mother naturally receives per hour
- β: half-life of contaminant
- δ_c: zero if time is prior to conception; one otherwise.
- δ_d: zero if time is prior to delivery; one otherwise.
- δ_b: zero if time is prior to the end of breastfeeding; one otherwise.

There are various other parameters not listed that appear in the nonconstant terms. For example, the mother's age and weight at conception.

The system described can be simplified to the following:

$$\begin{aligned}\frac{dM}{dt} &= \frac{d}{24}B - \frac{\ln 2}{\beta}M + [\delta_c - \delta_d]\left(\frac{C}{V_C} - \frac{M}{V_M}\right) - [\delta_d - \delta_b]\frac{ML}{V_M} \\ \frac{dC}{dt} &= -[\delta_c - \delta_d]\left(\frac{C}{V_C} - \frac{M}{V_M}\right) - \delta_d C\frac{\ln 2}{\beta} + [\delta_d - \delta_b]\frac{ML}{V_M}.\end{aligned} \tag{1}$$

Time is measured in hours, concentration units are in μg, and volume in kg.

Once the child is delivered and breastfeeding is over, (1) reduces to the linear system

$$\begin{aligned}\frac{dM}{dt} &= \frac{d}{24}B - \frac{\ln 2}{\beta}M \\ \frac{dC}{dt} &= -C\frac{\ln 2}{\beta}.\end{aligned} \tag{2}$$

It is evident that the child will immediately begin clearing contaminants, eventually clearing virtually all of them. The behavior of contaminants in the mother is governed by a linear equation whose solution depends on $B(t)$.

A simulation of (1) is shown in Figure 1. Parameter values were chosen based on values found in the `acslX` code of Verner et al. [2013].

Table 1: Parameter values for the simulation shown in Figure 1.

Parameter	Value	Parameter	Value
Mother age	30 years	β	5000 hours
Conception weight	58.0	Conception time	6 months
Pre-pregnancy weight	58.0	Delivery time	15 months
d	5 μg	Breastfeeding end	27 months
V_M	.2	$L(t)$	10^{-4}
V_C	.1		

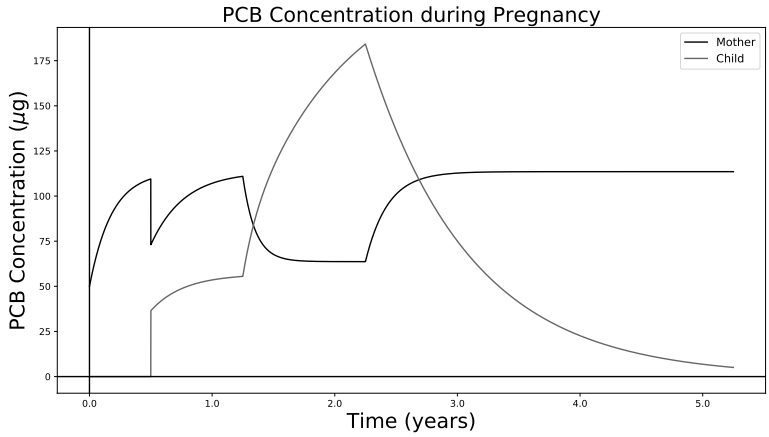

Figure 1: Partially-complete simulation of (1) with initial conditions guessed by reading the `acslX` code of Verner et al. [2013]. Some nonconstant terms (in particular, $B(t)$) are treated as constant in this simulation for simplicity, but would be truly nonconstant in the final version.

References

Marc-André Verner, Dean Sonneborn, Kinga Lancz, Gina Muckle, Pierre Ayotte, Éric Dewailly, Anton Kocan, Lubica Palkovicová, Tomas Trnovec, Sami Haddad, et al. Toxicokinetic modeling of persistent organic pollutant levels in blood from birth to 45 months of age in longitudinal birth cohort studies. 2013.