Number Theory Homework VI

RDB

July 9, 2022

This is an ungraded *bonus* assignment. We covered a lot of material this week, so we need to see some problems before the final.

Exercise 1 Find all the primitive roots mod 3, 5, 7, and 11.

Exercise 2 Write the orders of every integer mod 11.

Exercise 3 How many primitive roots does 101 have?

Exercise 4 Compute

$$(\phi * \mu)(10)$$

Exercise 5 Prove that fg is multiplicative if f and g are.

Exercise 6

(**a**) Why is $\phi * \mu^2$ multiplicative?

(**b**) Evaluate $(\phi * \mu^2)(p^k)$ for a prime power p^k.

Exercise 7 What are the quadratic residues mod 13?

Exercise 8 Prove that ab is a quadratic residue mod an odd prime p iff either:

1. a and b are both quadratic residues mod p; or
2. neither a nor b is a quadratic residue mod p.

[Hint: Euler's criterion.]