Who wants to be an analyst?

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November 20, 2021

Format

Teams of 3-4 people.

Every team starts with 10 points.

You *wager* points.

Questions are multiple choice. Some have more than one answer.
You must choose all correct answers.

Winner gets their choice of candies.

Example question

If $\lim_{n\to\infty} a_n = 5$, then...

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If $\lim_{n\to\infty} a_n = 5$, then…

(**a**) $a_n > 0$ for all n

(**b**) $a_n > 0$ for all but finitely many n

(**c**) $a_n = 5$ for infinitely many n

(**d**) $a_n \leq 6$ for all but finitely many n.

Functional limits

Let $f\colon \mathbf{R} \to \mathbf{R}$ be a function such that $\{f(x_n)\}$ is Cauchy for every $x_n \to x_0$ with $x_n \neq x_0$. Then $f\ldots$

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(**a**) is continuous at x_0.

(**b**) has a limit at x_0.

(**c**) is Cauchy at x_0.

(**d**) is positive at x_0.

Answer: (b).

If $\lim_{x \to x_0} f(x) = L$, then…

If $\lim_{x \to x_0} f(x) = L$, then...

(a) $f(x_0) = L$

(b) $f(x) < L$ for all x in a neighborhood of x_0

(c) $f(x) = L$ for some x in every neighborhood of x_0.

(d) x_0 is an accumulation point of $\{x : |f(x) - L| < 1\}$.

Answer: (d)

If $f\colon [a,b] \to \mathbf{R}$ is monotonically increasing, then…

If $f\colon [a,b] \to \mathbf{R}$ is monotonically increasing, then...

(**a**) f has at most countably many discontinuities.

(**b**) f is continuous.

(**c**) $f(b) > f(a)$

(**d**) For every $f(a) < y < f(b)$, there exists some some $x \in [a,b]$ such that $f(x) = y$.

Answer: (a)

$$\lim_{x \to 1} (???)$$

Answer: (c)

$$\lim_{x \to 1} \frac{x^2 - 2x + 1}{x - 1} = \ldots$$

(**a**) -2

(**b**) -1

(**c**) 0

(**d**) 1

(**e**) 2

Answer: (c)

Give an example of a bounded function on $[0, 1]$ that has a limit at…

Give an example of a bounded function on $[0, 1]$ that has a limit at. . . exactly *one* point.

Answer:

$$f(x) = \begin{cases} x & x \in \mathbf{Q} \\ 0 & x \notin \mathbf{Q} \end{cases}$$

If f is a function on a finite set, then…

If f is a function on a finite set, then. . .

(**a**) f is not continuous

(**b**) f is bounded

(**c**) f is continuous

(**d**) f has a limit everywhere

Answer: (b), (c)

If $f\colon \mathbf{R} \to \mathbf{R}$ is continuous and $f(x_0) > 0$, then...

If $f\colon \mathbf{R} \to \mathbf{R}$ is continuous and $f(x_0) > 0$, then...

(**a**) $f(x) > 0$ for all $x \in \mathbf{R}$

(**b**) $f(x) > 0$ for all x in some neighborhood of x_0

(**c**) For every $\epsilon > 0$, there exists some x such that $|f(x) - f(x_0)| > \epsilon$.

(**d**) x_0 may be an accumulation point of $\{x : f(x) = 0\}$.

Answer: (b)

Topology

A set E is open iff…

A set E is open iff...

(**a**) For every $x \in E$, there exists an $\epsilon > 0$ such that $(x - \epsilon, x + \epsilon) \subseteq E$.

(**b**) For every $x \in E$, for all $\epsilon > 0$ we have $(x - \epsilon, x + \epsilon) \subseteq E$.

(**c**) For every $x \in E$, there exists a neighborhood N of x such that $E \subseteq N$.

(**d**) For every $x \in E$, every neighborhood of x contains points of E.

Answer: (a)

The [ ] theorem says that E is compact iff it is closed and bounded.

The [   ] theorem says that E is compact iff it is closed and bounded.

(**a**) Completeness
(**b**) Heine–Borel
(**c**) Compactness
(**d**) Riemann–Gauss

Answer: (b)

If f is a function on a compact set K, then…

If f is a function on a compact set K, then...

(**a**) $f^{-1}(E)$ is open whenever E is open

(**b**) $f^{-1}(E)$ is bounded for all E

(**c**) $f^{-1}(E)$ is closed for all E

(**d**) $f^{-1}(E)$ is compact if E is compact

Answer: (b)

Choose every open set:

Choose every open set:

(**a**) $\mathbf{Q}$

(**b**) $\bigcup_{k\geq 1}[-k, k]$

(**c**) $(0, 1)$

(**d**) $\bigcap_{k\geq 1}[-1 - 1/k, 1 + 1/k]$

(**e**) $\{x : x^2 < 10\}$

Answer: (b), (c), (d)

If $\{E_\alpha\}$ is a collection of open sets, then…

If $\{E_\alpha\}$ is a collection of open sets, then...

(**a**) $\bigcup_\alpha E_\alpha$ is infinite

(**b**) $\bigcup_\alpha E_\alpha$ is open

(**c**) $\bigcup_\alpha E_\alpha$ is not closed

(**d**) $\bigcup_\alpha E_\alpha$ may be closed

Answer: (b), (d)

If f is a function on a compact set K, then…

If f is a function on a compact set K, then...

(**a**) f is bounded.

(**b**) f has a limit at at least one $x_0 \in K$.

(**c**) f is continuous.

(**d**) f has a maximum on K.

Answer: None

Since $[0, 1]$ is closed…

Since $[0, 1]$ is closed...

(**a**) $(-\infty, 0) \cup (1, \infty)$ is open.

(**b**) $[0, 1]$ is not open.

(**c**) $(-\infty, 0) \cup (1, \infty)$ has no accumulation points.

Answer: (a)

If $E_1, \ldots, E_n$ are closed, then…

If $E_1, \ldots, E_n$ are closed, then...

(**a**) $\bigcup_{k=1}^{n} E_k$ is closed

(**b**) $\bigcup_{k=1}^{n} E_k$ is open

(**c**) $\bigcup_{k=1}^{n} E_k$ is not open

(**d**) $\bigcup_{k=1}^{n} E_k$ is compact

Answer: (a)

If $E_1, \ldots, E_n$ are closed, and E_1 is bounded then...

(**a**) $\bigcap_{k=1}^{n} E_k$ is infinite

(**b**) $\bigcap_{k=1}^{n} E_k$ is open

(**c**) $\bigcap_{k=1}^{n} E_k$ is not open

(**d**) $\bigcap_{k=1}^{n} E_k$ is compact

Answer: (d)

A set K is compact iff...

A set K is compact iff...

(**a**) K is contained in finitely many open sets.

(**b**) For every collection of open sets $\{U_\alpha\}$ such that $K \subseteq \bigcup_\alpha U_\alpha$, there exist finitely many open sets $O_1, \ldots, O_n$ such that $K \subseteq \bigcup_{K=1}^{n} O_k$.

(**c**) Every open cover of K contains a finite subcover.

(**d**) K is not open and bounded.

Answer: (c)

If $f\colon \mathbf{R} \to \mathbf{R}$ is continuous, then...

(**a**) $f^{-1}(E)$ is infinite whenever E is infinite

(**b**) $f^{-1}(E)$ is finite whenever E is finite

(**c**) $f^{-1}(E)$ is compact whenever E is compact

(**d**) $f^{-1}(E)$ is open whenever E is open

Answer: (d)

Let $f\colon [0,1] \to \mathbf{R}$ be continuous and satisfy

$$f(0) = 0 \qquad f(1) = 1.$$

Then…

Let $f\colon [0,1] \to \mathbf{R}$ be continuous and satisfy

$$f(0) = 0 \qquad f(1) = 1.$$

Then...

(**a**) $f(c) = 1/2$ for some $c \in (0,1)$.

(**b**) $\{x : f(x) = 0\}$ has an accumulation point in $[0,1]$.

(**c**) $f(c) = 1/2$ for *exactly one* $c \in (0,1)$.

(**d**) $0 < f(x) < 1$ for $x \in (0,1)$.

Answer: (a)